A QSO Generator

An upside-down unicycle rotates simultaneously on the vertical and horizontal axes. A red patch on the tire traces a Quasi-Spherical Orbit on the unit sphere.

Reference:

QSO - The Mathematics and Physics of Quasi-Spherical Orbits by Robert G. Chester (2009). ISBN 978-0-9840727-0-5.

Software:

This file is written for Graphing Calculator 3.2 by Pacific Tech <http://www.PacificT.com/>.

The QSO ratio:

a = rotation around the z-axis (equatorial orbit)

b = rotation around the y-axis (polar orbit)

Change the ratio to generate different QSOs.

$$a = 1, b = 1$$

Clone curves and axes are added to give a thicker, brighter line for printing. For viewing on the screen they are not needed. To turn off the clones, set f = 0.

Separation of clone curves from the original:

$$f = 0.007, X = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}, Y = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}, Z = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$$

The angle in radians corresponding to the fraction of a complete orbit:

$$g = 2\pi n$$

A parametric hemisphere which stands in for the unit sphere:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.99 \begin{bmatrix} \cos\left(\frac{7\pi}{4}\right) & -\sin\left(\frac{7\pi}{4}\right) & 0 \\ \sin\left(\frac{7\pi}{4}\right) & \cos\left(\frac{7\pi}{4}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-60°) & -\sin(-60°) \\ 0 & \sin(-60°) & \cos(-60°) \end{bmatrix} \begin{bmatrix} \sin\frac{u\pi}{2} \cdot \cos 2\pi v \\ \sin\frac{u\pi}{2} \cdot \sin 2\pi v \\ \cos\frac{u\pi}{2} \end{bmatrix}$$

Radius of the spokes:

$S = 0.04$

On-Off switches:

On = 1

Off = 0

The QSO

$Q = 1$

Unicycle

$U = 1$

Axes

$A = 1$

Meridians

$M = 0$

Equator

$E = 1$

The QSO:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin b g t)(\cos a g t) \\ (\sin b g t)(\sin a g t) \\ \cos b g t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin b g t)(\cos a g t) \\ (\sin b g t)(\sin a g t) \\ \cos b g t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin b g t)(\cos a g t) \\ (\sin b g t)(\sin a g t) \\ \cos b g t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin b g t)(\cos a g t) \\ (\sin b g t)(\sin a g t) \\ \cos b g t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt)\,(\cos agt) \\ (\sin bgt)\,(\sin agt) \\ \cos bgt \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt)\,(\cos agt) \\ (\sin bgt)\,(\sin agt) \\ \cos bgt \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt)\,(\cos agt) \\ (\sin bgt)\,(\sin agt) \\ \cos bgt \end{bmatrix} - Z$$

A small sphere that rotates in sync with the wheel:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.075\,U \begin{bmatrix} \sin 2\pi bu \cdot \cos 2\pi av \\ \sin 2\pi bu \cdot \sin 2\pi av \\ \cos 2\pi bu \end{bmatrix} + 0.925 \begin{bmatrix} (\sin bg)\,(\cos ag) \\ (\sin bg)\,(\sin ag) \\ \cos bg \end{bmatrix}$$

Spokes:

First term: rotation around z.

Second term: rotation around y.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-bg) & 0 & -\sin(-bg) \\ 0 & 1 & 0 \\ \sin(-bg) & 0 & \cos(-bg) \end{bmatrix} 0.925 \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-bg) & 0 & -\sin(-bg) \\ 0 & 1 & 0 \\ \sin(-bg) & 0 & \cos(-bg) \end{bmatrix} 0.925 \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ -v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-bg + 60^\circ) & 0 & -\sin(-bg + 60^\circ) \\ 0 & 1 & 0 \\ \sin(-bg + 60^\circ) & 0 & \cos(-bg + 60^\circ) \end{bmatrix} 0.925 \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ 2v - 1 \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-bg + 120^\circ) & 0 & -\sin(-bg + 120^\circ) \\ 0 & 1 & 0 \\ \sin(-bg + 120^\circ) & 0 & \cos(-bg + 120^\circ) \end{bmatrix} 0.925 \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ 2v - 1 \end{bmatrix} U$$

Wheel:

(Equation modified from *Learning Math*, pp. 84-88.)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (\cos 2\pi u)\,(1 + 0.1\cos(2\pi v)) \\ 0.1\sin(2\pi v) \\ (\sin 2\pi u)\,(1 + 0.1\cos(2\pi v)) \end{bmatrix} 0.9\,U$$

Hub (two halves):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1\,(\cos 2\pi u)\,(1 + \cos(2\pi v)) \\ 0.05\sin\pi v \\ 0.1\,(\sin 2\pi u)\,(1 + \cos(2\pi v)) \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1\,(\cos 2\pi u)\,(1 + \cos(2\pi v)) \\ -0.05\sin\pi v \\ 0.1\,(\sin 2\pi u)\,(1 + \cos(2\pi v)) \end{bmatrix} U$$

The fork:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.145\,(0.28\sin(2\pi v)) \\ 0.28\,(-\cos\pi u)\,(1 + 0.145\,(\cos(2\pi v))) \\ 0.28\,(-\sin\pi u)\,(1 + 0.145\,(\cos(2\pi v))) - 1 \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S\cos 2\pi u \\ (S\sin 2\pi u) - 0.28 \\ -v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S\cos 2\pi u \\ (S\sin 2\pi u) + 0.28 \\ -v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = U \begin{bmatrix} S\sin 2\pi u \\ S\cos 2\pi u \\ 0.6v - 1.9 \end{bmatrix}$$

The axle:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S\cos 2\pi u \\ 0.55\,v - 0.275 \\ S\sin 2\pi u \end{bmatrix} U$$

Small spheres to give axle a finished look:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = S \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin 2\pi bu \cdot \sin 2\pi av \\ \cos 2\pi bu + 7 \\ \sin 2\pi bu \cdot \cos 2\pi av \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = S \begin{bmatrix} \cos ag & -\sin ag & 0 \\ \sin ag & \cos ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin 2\pi bu \cdot \sin 2\pi av \\ \cos 2\pi bu - 7 \\ \sin 2\pi bu \cdot \cos 2\pi av \end{bmatrix} U$$

Axes:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} - Z$$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} - Z$$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 0.98t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 0.98t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 0.98t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ 0.98t \end{bmatrix} - Y$$

Great circle meridians:

yz meridian:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - Z$$

xz meridian:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = M \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - Z$$

xy meridian (equator):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E\begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E\begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E\begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = E\begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - Z$$

Alignment circles:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 45° & \cos 45° & 0 \\ \sin 45° & \sin 45° & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \sin 2\pi t \\ 0 \\ \cos 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 40° & \cos 40° \\ 0 & \sin 40° & \sin 40° \end{bmatrix}\begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \cos 40° & \cos 40° \\ 1 & 0 & 0 \\ 0 & \sin 40° & \sin 40° \end{bmatrix}\begin{bmatrix} \sin 2\pi t \\ 0 \\ \cos 2\pi t \end{bmatrix}$$