

The QSO ratio:

a = rotation around the z-axis (longitude)

b = rotation around the y-axis (latitude)

$$a = 1, b = 1$$

Separation of duplicate curves from the original:

$$f = 0.007, X = \begin{bmatrix} f \\ 0 \\ 0 \end{bmatrix}, Y = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}, Z = \begin{bmatrix} 0 \\ 0 \\ f \end{bmatrix}$$

The angle in radians corresponding to the fraction of a complete orbit:

$$g = 2\pi n$$

The parametric hemisphere:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.99 \begin{bmatrix} \cos\left(\frac{7\pi}{4}\right) - \sin\left(\frac{7\pi}{4}\right) & 0 \\ \sin\left(\frac{7\pi}{4}\right) & \cos\left(\frac{7\pi}{4}\right) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-60^\circ) & -\sin(-60^\circ) \\ 0 & \sin(-60^\circ) & \cos(-60^\circ) \end{bmatrix} \begin{bmatrix} \sin\frac{u\pi}{2} \cdot \cos 2\pi v \\ \sin\frac{u\pi}{2} \cdot \sin 2\pi v \\ \cos\frac{u\pi}{2} \end{bmatrix}$$

The radius of the virtual sphere on which the curves are graphed:

$$R = 1$$

Radius of the spokes:

$$S = 0.04$$

Note: For unicycle, S = 0.04.

For isometric view without unicycle, S = 0.03

On-Off switches:

$$\text{On} = 1$$

$$\text{Off} = 0$$

QSO

Axes

Meridians

Unicycle

The linear QSOrbit:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = Q \begin{bmatrix} (\sin bgt) (\sin agt) \\ (\sin bgt) (\cos agt) \\ \cos bgt \end{bmatrix} - Z$$

A small sphere that rotates in sync with the wheel:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0.075 R U \begin{bmatrix} \sin 2\pi b u - \cos 2\pi a v \\ \sin 2\pi b u - \sin 2\pi a v \\ \cos 2\pi b u \end{bmatrix} + 0.925 R \begin{bmatrix} (\sin b g) (\sin a g) \\ (\sin b g) (\cos a g) \\ \cos b g \end{bmatrix}$$

Note: For unicycle, use 0.925R.

For isometric view without unicycle, use R.

Spokes:

First term: rotation around z.

Second term: rotation around y.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-b g) & 0 & -\sin(-b g) \\ 0 & 1 & 0 \\ \sin(-b g) & 0 & \cos(-b g) \end{bmatrix} 0.95 R \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-b g) & 0 & -\sin(-b g) \\ 0 & 1 & 0 \\ \sin(-b g) & 0 & \cos(-b g) \end{bmatrix} 0.95 R \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ -v \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-b g + 60^\circ) & 0 & -\sin(-b g + 60^\circ) \\ 0 & 1 & 0 \\ \sin(-b g + 60^\circ) & 0 & \cos(-b g + 60^\circ) \end{bmatrix} 0.95 R \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ 2v - 1 \end{bmatrix} U$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-b g + 120^\circ) & 0 & -\sin(-b g + 120^\circ) \\ 0 & 1 & 0 \\ \sin(-b g + 120^\circ) & 0 & \cos(-b g + 120^\circ) \end{bmatrix} 0.95 R \begin{bmatrix} S \sin 2\pi u \\ S \cos 2\pi u \\ 2v - 1 \end{bmatrix} U$$

Wheel:

(Equation modified from Learning Math, pp. 84-88.)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} 0.9 R \begin{bmatrix} (\cos 2\pi u) (1 + 0.1 \cos(2\pi v)) \\ 0.1 \sin(2\pi v) \\ (\sin 2\pi u) (1 + 0.1 \cos(2\pi v)) \end{bmatrix} U$$

Hub (two halves):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 (\cos 2\pi u) (1 + \cos (2\pi v)) \\ 0.05 \sin \pi v \\ 0.1 (\sin 2\pi u) (1 + \cos (2\pi v)) \end{bmatrix} \mathbf{U}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 (\cos 2\pi u) (1 + \cos (2\pi v)) \\ -0.05 \sin \pi v \\ 0.1 (\sin 2\pi u) (1 + \cos (2\pi v)) \end{bmatrix} \mathbf{U}$$

The fork:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{bmatrix} 0.145 (0.28 \sin (2\pi v)) \\ 0.28 (-\cos \pi u) (1 + 0.145 (\cos (2\pi v))) \\ 0.28 (-\sin \pi u) (1 + 0.145 (\cos (2\pi v))) - 1 \end{bmatrix} \mathbf{U}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{bmatrix} S \cos 2\pi u \\ (S \sin 2\pi u) - 0.28 \\ -v \end{bmatrix} \mathbf{U}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{bmatrix} S \cos 2\pi u \\ (S \sin 2\pi u) + 0.28 \\ -v \end{bmatrix} \mathbf{U}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R} \mathbf{U} \begin{bmatrix} S \cos 2\pi u \\ S \sin 2\pi u \\ 0.6v - 1.9 \end{bmatrix}$$

The axle:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{R} \begin{bmatrix} S \cos 2\pi u \\ 0.55v - 0.275 \\ S \sin 2\pi u \end{bmatrix} \mathbf{U}$$

Small spheres to give axle a finished look:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{R} \mathbf{S} \begin{bmatrix} \sin ag & \cos ag & 0 \\ \cos ag & -\sin ag & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin 2\pi bu - \sin 2\pi av \\ \cos 2\pi bu + 7R \\ \sin 2\pi bu - \cos 2\pi av \end{bmatrix} \mathbf{U}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = RS \begin{bmatrix} \sin a g & \cos a g & 0 \\ \cos a g & -\sin a g & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin 2\pi b u - \sin 2\pi a v \\ \cos 2\pi b u - 7R \\ \sin 2\pi b u - \cos 2\pi a v \end{bmatrix} U$$

Axes:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 2t - 1 \\ 0 \\ 0 \end{bmatrix} - Z$$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 2t - 1 \\ 0 \end{bmatrix} - Z$$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix} - Y$$

Great circle meridians:

xz meridian:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} 0 \\ \cos 2\pi t \\ \sin 2\pi t \end{bmatrix} - Z$$

yz meridian:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ 0 \\ \sin 2\pi t \end{bmatrix} - Z$$

xy meridian (equator):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - X$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - Y$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} + Z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = MR \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix} - Z$$

Alignment circles:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \cos 45^\circ & 0 \\ \sin 45^\circ & \sin 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sin 2\pi t \\ 0 \\ \cos 2\pi t \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 40^\circ & \cos 40^\circ \\ 0 & \sin 40^\circ & \sin 40^\circ \end{bmatrix} \begin{bmatrix} \cos 2\pi t \\ \sin 2\pi t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & \cos 40^\circ & \cos 40^\circ \\ 1 & 0 & 0 \\ 0 & \sin 40^\circ & \sin 40^\circ \end{bmatrix} \begin{bmatrix} \sin 2\pi t \\ 0 \\ \cos 2\pi t \end{bmatrix}$$

